Ω *and*Φ *inUNNSStructuralRecursion*

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Abstract

Phase–F of the UNNS recursion framework establishes the micro–recursive mechanics of the PE–27 operator family and exposes the fundamental closure failures of raw recursion: idempotence drift, flux imbalance, and reversibility loss. Phase–G extends the recursion into the structural regime by introducing two new mathematical components: the closure correction operator Ω, and the nonlinear operator manifold Φ.

The Ω–operator enforces the necessary closure conditions for stable recursion, while the Φ–stack reconstructs the non–linear dependency structure between the elementary UNNS operators. Together, these transform the recursion from a diagnostic engine into a structurally self–consistent predictive substrate.

Introduction

The UNNS Substrate is built on the principle that nonlinear recursion is the fundamental generative mechanism underlying structure, invariants, and observable quantities. The micro–recursion engine PE–27, used in Phase–F, revealed essential operator behaviors but lacked the structural properties required for stable prediction: idempotence, reversibility, and flux neutrality. As a consequence, raw recursion $R(\tau)$ produced drifting fields, unstable operator outputs, and highly sensitive predictions.

Phase–G introduces the first major structural extension of the UNNS recursion framework. Two mathematical constructs are added to the recursion: the closure operator Ω and the nonlinear operator manifold Φ. The Ω operator corrects structural deviations in real time, driving the recursion toward a fixed point that satisfies all closure criteria. The Φ manifold encodes nonlinear interactions among operator outputs, enabling stable, well–posed prediction equations for observables such as Λ, σ_8, r_{drag}, and $\mu_{p/e}$.

The purpose of this monograph is to provide a complete and rigorous development of PE–27G, the Ω–corrected, Φ–enhanced structural recursion engine. The material is organized into four major thematic components:

- **Foundational Structure (Appendices A–F).** These chapters define the PE–27 recursion, introduce the closure operator Ω, and establish the fixed point theory, spectral properties, and stability results that justify Phase–G.
- **Geometric and Information–Theoretic Structure (Appendices G–Q).** These chapters develop the quadratic geometry of the Φ manifold, operator coupling theory, entropy flow analysis, collapse analogues with Operator XII, and the emergence of invariants.
- **Phase Space, Stochasticity, and Analytical Methods (Appendices R–U).** These chapters analyze the geometric phase space of PE–27G, noise effects, analytic approximations of the corrected map, and the role of micro–folding in operator emergence.
- **Advanced Dynamics, Probability Structure, and Open Questions (Appendices V–Z).** These chapters cover bifurcation structure, invariant measures, high–dimensional operator geometry, numerical convergence results, and open problems for future research.

Collectively, these 26 appendices establish PE–27G as a structurally predictive recursion engine capable of generating stable invariants through nonlinear dynamics. Phase–G represents the first stage in which UNNS recursion becomes mathematically closed, dynamically stable, and geometrically interpretable. It provides the theoretical and computational foundation required for subsequent phases of UNNS structural development.

0.1 2. Preliminaries

Let $\tau(x)$ denote the τ–field on a discrete lattice. Let R be a single recursion application of PE–27, so that

$$\tau_{n+1} = R(\tau_n).$$

Let O_i denote the scalar operator features extracted from the τ–field after recursion:

$$O_{13},\ O_{14},\ O_{15},\ O_{16},\ O_{21}.$$

In Phase–F the prediction equations depend linearly on O_i. However, operator–diagnostic analysis shows that several observables require nonlinear dependencies such as $O_{13}O_{16}$ and O_{21}^2. Phase–G therefore augments PE–27 with two mathematical structures: the closure operator Ω and the nonlinear operator stack Φ.

0.2 3. The Need for Structural Correction

Raw recursion R fails three closure conditions:

(i) Idempotence

$$\Delta_{C1} = R(\tau) - \tau \neq 0.$$

(ii) Flux Neutrality

Let $F = \nabla\tau$. Then

$$\Delta_{C3} = \nabla \cdot \nabla\tau \neq 0.$$

(iii) Reversibility

$$\Delta_{C5} = R^{-n}(R^n(\tau)) - \tau \neq 0.$$

These deviations propagate into observable predictions and prevent stability.

0.3 4. Definition of the Closure Operator Ω

Define the closure–corrected field by

$$\Omega[\tau] = \tau - \eta_1 \Delta_{C1} - \eta_2 \Delta_{C3} - \eta_3 \Delta_{C5}.$$

The updated recursion becomes:

$$\tau_{n+1} = \Omega(R(\tau_n)).$$

The coefficients η_i are small positive constants controlling the strength of the correction.

0.4 5. The Nonlinear Operator Manifold Φ

Let

$$O = (O_{13}, O_{14}, O_{15}, O_{16}, O_{21})^T.$$

Define Φ as

$$\Phi = W^T O + O^T Q O,$$

where W is a weight vector and Q is a symmetric matrix encoding pairwise operator couplings.

Explicitly:

$$\Phi = \sum_i W_i O_i + \sum_{i<j} Q_{ij} O_i O_j.$$

5.1 Role in Predictions

An observable X is predicted via

$$\hat{X} = f(O_1, \ldots, O_5, \Phi).$$

Examples:

$$\Lambda = \Lambda_0 \exp(a\Phi + bO_{13}),$$

$$\sigma_8 = \sigma_{8,0} + k_1 \Phi + k_2 O_{21},$$

$$r_{\text{drag}} = A + B\Phi + C O_{13} O_{16},$$

$$\mu_{p/e} = m_0 \exp(k_1 O_{21} + k_2 O_{16} O_{21}).$$

0.5 6. Interaction Between Ω and Φ

The recursion architecture becomes:

$$\tau_{n+1} = \Omega(R(\tau_n)),$$

$$O_i = O_i(\tau_{n+1}),$$
$$\Phi = W^T O + O^T Q O,$$
$$\hat{X} = f(O, \Phi).$$

Thus Ω stabilizes the field while Φ reconstructs the nonlinear operator interaction structure.

0.6 7. Summary

Phase–G introduces two essential components:

1. The closure operator Ω, which enforces structural convergence by correcting deviations in idempotence, flux neutrality, and reversibility.
2. The nonlinear operator manifold Φ, which captures the multi–operator dependencies required for predicting observables such as Λ, σ_8, r_{drag}, and $\mu_{p/e}$.

Together these convert PE–27 from a diagnostic recursion into a structurally predictive substrate.

Appendix A: The PE–27 Micro–Recursion Engine

For completeness we summarize the mathematical structure of the PE–27 recursion operator used throughout this work. Let $\tau_n(x)$ denote the τ–field after n recursion steps on a discrete lattice $\mathcal{L}$. A single PE–27 update consists of six sub–steps:

A.1. Diffusive Update

A discrete Laplacian Δ is applied to generate a smoothing term:

$$\tau^{(1)} = \tau_n + \lambda\, \Delta \tau_n,$$

where $\lambda > 0$ is the coupling parameter.

A.2. Torsion Kernel Transformation

Let $T_r[\tau]$ denote the torsion operator with kernel radius r. This produces a local curvature–torsion response:

$$\tau^{(2)} = \tau^{(1)} + \alpha_c\, T_r[\tau^{(1)}],$$

where α_c is the closure–strength parameter.

A.3. Nonlinear Micro–Folding

A nonlinear contraction map F is applied:

$$\tau^{(3)} = F(\tau^{(2)}),$$

used to model micro–scale folding and local feature amplification.

A.4. Sealing Event

Every s steps, for a sealing interval $s \in N$, a projection toward local closure is applied:

$$\tau^{(4)} = \{\ S\ (\tau^{(3)}), n \equiv 0s, \tau^{(3)}, otherwise.$$

A.5. Noise Injection

A stochastic term models micro–scale fluctuation:

$$\tau^{(5)} = \tau^{(4)} + \sigma\, \xi_n,$$

where each ξ_n is an independent random field with zero mean.

A.6. Normalization

A final normalization step produces the next field:

$$\tau_{n+1} = \mathcal{N}(\tau^{(5)}),$$

where $\mathcal{N}$ rescales the field to a fixed range.

The full PE–27 recursion is thus:

$$R(\tau_n) = \mathcal{N}(\tau_n + \lambda\, \Delta\tau_n + \alpha_c\, T_r[\tau_n] + \sigma\, \xi_n),$$

with sealing and nonlinear folding applied at the appropriate stages.

Appendix B: Operator Feature Extraction

The operator outputs used in the Φ–stack are defined as follows.

$$O_{13} = meancurvaturegradientmagnitude,$$

$$O_{14} = localfoldingtension,$$

$$O_{15} = variancereductionindex,$$

$$O_{16} = closure--torsionenergy,$$

$$O_{21} = micro--torsiondensity.$$

Each O_i is a scalar computed from τ_{n+1} and forms the input vector O used in the nonlinear manifold:

$$\Phi = W^T O + O^T Q O.$$

This appendix establishes the mathematical basis upon which the Ω and Φ structures operate.

Appendix C: Properties of Ω and Φ Under Repeated Recursion

This appendix summarizes the mathematical behavior of the closure operator Ω and the nonlinear manifold Φ under iterated application of the structural recursion

$$\tau_{n+1} = \Omega(R(\tau_n)).$$

C.1. Contraction of Closure Residuals

Let the closure residual vector be

$$\Delta(\tau) = (\Delta_{C1}(\tau),\, \Delta_{C3}(\tau),\, \Delta_{C5}(\tau)).$$

Because Ω subtracts a linear combination of the residuals,

$$\Omega[\tau] = \tau - M\Delta(\tau),$$

where M is a diagonal matrix with positive entries

$$M = \mathrm{diag}(\eta_1, \eta_2, \eta_3),$$

the residual evolves as

$$\Delta(\tau_{n+1}) = \Delta(\Omega(R(\tau_n))).$$

Under mild smoothness assumptions on R, the Jacobian of this map has spectral radius strictly less than 1 when the parameters η_i are sufficiently small. Thus

$$\|\Delta(\tau_{n+1})\| < \|\Delta(\tau_n)\|.$$

Therefore:

Result C.1.

Repeated recursion drives the field toward closure,

$$\Delta(\tau_n) \to 0,$$

and hence τ_n approaches a structurally stable fixed point.

C.2. Asymptotic Idempotence

Because closure implies $R(\tau^*) = \tau^*$, convergence of the residuals guarantees:

Result C.2.

If $\tau_n \to \tau^*$ under the Ω–corrected recursion, then τ^* is an idempotent point of R:

$$R(\tau^*) = \tau^*.$$

Thus Ω converts a generally non-idempotent map R into a sequence that converges to one of its idempotent points.

C.3. Boundedness and Stability of Φ

Let $O_i(\tau)$ denote the operator outputs defined in Appendix B. Since Φ is a quadratic polynomial in the components of O, we write

$$\Phi(\tau) = W^T O(\tau) + O(\tau)^T Q \, O(\tau).$$

As τ_n approaches its fixed point τ^*, continuity of the operator extraction maps implies

$$O(\tau_n) \to O(\tau^*), \qquad \Phi(\tau_n) \to \Phi(\tau^*).$$

Result C.3.

The nonlinear manifold Φ is asymptotically constant:

$$\Phi_n := \Phi(\tau_n) \quad satisfies \quad \Phi_n \to \Phi^*.$$

Thus Φ becomes a stable invariant of the recursion once closure is achieved.

C.4. Predictive Stability of Observables

For any observable X represented in Phase–G by

$$\hat{X} = f(O(\tau), \Phi(\tau)),$$

convergence of both $O(\tau_n)$ and $\Phi(\tau_n)$ implies:

Result C.4.

The predicted observables converge:

$$\hat{X}_n \to \hat{X}^*, \qquad where \quad \hat{X}^* = f(O(\tau^*), \Phi(\tau^*)).$$

Thus Phase–G recursion yields stable predictions once Ω has driven the system to a fixed point.

C.5. Decoupling of Field Stabilization and Operator Dynamics

An essential structural feature of Phase–G is that Ω acts on the field τ, while Φ acts on the operator space of derived features. Because τ_n converges before the nonlinear manifold is evaluated, we obtain the decoupling principle:

Result C.5.

Field convergence under Ω implies convergence of all operator features, and hence of the manifold Φ.

This separation ensures modularity of the recursion architecture.

C.6. Summary of Recursion Properties

Under repeated recursion:

- τ_n converges to a closure–consistent fixed point,
- Φ_n converges to a nonlinear invariant of that fixed point,
- predicted observables stabilize to well-defined values,
- structural drift is removed by Ω,
- multi-operator coupling is incorporated through Φ.

Together, these results establish that the Ω–corrected, Φ–enhanced recursion of Phase–G is both stable and structurally predictive.

Appendix D: Fixed Point Structure and Stability Analysis of PE–27G

The Phase–G recursion combines the PE–27 engine with the closure operator Ω:

$$\tau_{n+1} = \Omega(R(\tau_n)).$$

A fixed point τ^* satisfies

$$\tau^* = \Omega(R(\tau^*)).$$

D.1. Fixed Point Condition

Expanding the definition of Ω, the fixed point equation becomes

$$\tau^* = R(\tau^*) - \eta_1 \Delta_{C1}(\tau^*) - \eta_2 \Delta_{C3}(\tau^*) - \eta_3 \Delta_{C5}(\tau^*).$$

Since each $\Delta_{Ck}(\tau^*)$ measures deviation from closure, the fixed point must simultaneously satisfy:

$$\Delta_{C1}(\tau^*) = 0, \qquad \Delta_{C3}(\tau^*) = 0, \qquad \Delta_{C5}(\tau^*) = 0.$$

Therefore:

Result D.1.

A fixed point of the Phase–G recursion must satisfy

$$R(\tau^*) = \tau^*, \quad \nabla \cdot \nabla \tau^* = 0, \quad R^{-n}(R^n(\tau^*)) = \tau^*.$$

Thus τ^* is simultaneously idempotent, flux neutral, and reversible.

D.2. Linearization Around a Fixed Point

Let J_R denote the Jacobian of R at τ^*, and let J_Δ denote the Jacobian of the closure residual vector. Linearizing:

$$\tau_{n+1} - \tau^* = (J_R - MJ_\Delta)\,(\tau_n - \tau^*),$$

where

$$M = \mathrm{diag}(\eta_1, \eta_2, \eta_3).$$

Define the stability matrix

$$A = J_R - MJ_\Delta.$$

D.3. Stability Criterion

Let $\rho(A)$ be the spectral radius of A.

Result D.2.

The fixed point τ^* is stable if and only if

$$\rho(A) < 1.$$

Because M is positive diagonal, increasing η_i reduces $\rho(A)$, and thus Ω acts as a stabilizing perturbation of R.

D.4. Interpretation

The operator Ω shifts the fixed points of R into a stable set of idempotent, flux-neutral solutions. The stability of PE–27G depends on the competition between the intrinsic dynamics of R and the corrective strength of Ω.

Appendix E: Spectral Properties of the Closure Map

Let the closure residual be written in vector form:

$$\Delta(\tau) = (\,\Delta\,)_{C1}\,(\tau)\Delta_{C3}(\tau)\Delta_{C5}(\tau).$$

The closure map is

$$\Omega[\tau] = \tau - M\Delta(\tau), \qquad M = \mathrm{diag}(\eta_1, \eta_2, \eta_3).$$

E.1. Linearized Closure Map

Near a fixed point τ^*, expand:

$$\Delta(\tau) \approx J_\Delta(\tau - \tau^*).$$

Thus the linearized closure map is

$$\Omega[\tau] - \tau^* \approx (I - MJ_\Delta)(\tau - \tau^*).$$

E.2. Spectral Radius

Let the eigenvalues of J_Δ be $\lambda_1, \lambda_2, \lambda_3$. Then the eigenvalues of the linearized closure map are:

$$\mu_i = 1 - \eta_i \lambda_i.$$

Result E.1.

The closure map is contractive iff

$$|1 - \eta_i \lambda_i| < 1 \quad foreach \quad i.$$

This gives bounds:

$$0 < \eta_i < \frac{2}{\lambda_i}.$$

E.3. Relation to Stability of Full Recursion

Since Ω is composed with R, the effective spectral radius is that of

$$A = J_R - MJ_\Delta.$$

The closure map decreases the magnitude of eigenvalues corresponding to closure violations, and thereby decreases $\rho(A)$.

E.4. Asymptotic Spectrum

As $\Delta(\tau_n) \to 0$, the Jacobian of the combined map approaches

$$A^* = J_R(\tau^*),$$

and thus the recursion inherits asymptotic stability from the fixed point structure of R.

Result E.2.

Closure correction dominates in the transient phase, while PE–27's intrinsic spectrum governs long-term dynamics.

Appendix F: Example Recursion Fixed Points in 1D and 2D

To illustrate the behavior of PE–27G, we consider simplified examples on 1D and 2D lattices.

F.1. One–Dimensional Lattice

Let the field be defined on a 1D lattice:

$$\tau = (\tau_1, \tau_2, \ldots, \tau_N).$$

Under diffusion plus torsion, a fixed point satisfies:

$$\Delta\tau^* = 0, \qquad T_r[\tau^*] = 0.$$

The only solutions with periodic boundary conditions are constants:

$$\tau^* = c\mathbf{1}.$$

Applying closure conditions yields:

Result F.1.

In 1D, the PE–27G fixed point is a constant field.

F.2. Two–Dimensional Lattice

Let $\tau(x, y)$ be defined on a 2D grid. A closure–consistent fixed point must satisfy:

$$\Delta\tau^* = 0, \qquad \nabla \cdot \nabla\tau^* = 0, \qquad T_r[\tau^*] = 0.$$

Thus τ^* is harmonic:

$$\Delta\tau^* = 0.$$

With periodic boundary conditions, the solutions are Fourier modes:

$$\tau^*(x, y) = \sum_{k,\ell} a_{k\ell} \exp(2\pi i(kx + \ell y)),$$

where torsion constraints force $a_{k\ell} = 0$ unless $(k, \ell) = (0, 0)$.

Result F.2.

The only closure–consistent fixed point in 2D with periodic boundaries is a constant field.

F.3. Interpretation for the Full PE–27 Field

While these examples use simplified forms of the operators, they demonstrate the structural tendency of PE–27G: the recursion drives the system toward low–curvature, low–torsion equilibrium states. The full system retains small-scale structure through nonlinear micro–folding, but the closure constraints enforce global harmonicity.

Result F.3.

PE–27G produces a balance:

- nonlinear micro–scale structure from folding,
- global closure constraints from Ω,
- stable operator features feeding the Φ manifold.

This interaction yields the rich but stable fixed point behavior seen in numerical experiments.

Appendix G: Nonlinear Manifold Geometry of Φ

The nonlinear manifold Φ used in Phase–G is defined by

$$\Phi = W^T O + O^T Q O,$$

where $O = (O_{13}, O_{14}, O_{15}, O_{16}, O_{21})^T$ is the operator output vector and Q is a symmetric matrix encoding interactions.

G.1. Quadratic Manifold Structure

The scalar Φ defines a quadratic form on operator space R^5:

$$\Phi(O) = \sum_i W_i O_i + \sum_{i<j} Q_{ij} O_i O_j.$$

The level sets

$$\Phi(O) = c$$

are quadric surfaces (hyperboloids, ellipsoids, paraboloids) depending on the eigenstructure of Q.

G.2. Gradient of Φ

The gradient determines how Φ responds to perturbations in the operators:

$$\nabla_O \Phi = W + 2QO.$$

Thus the sensitivity of Φ to each operator O_i is:

$$\frac{\partial \Phi}{\partial O_i} = W_i + 2 \sum_j Q_{ij} O_j.$$

Result G.1.

The manifold geometry amplifies operators whose corresponding coefficients in Q are large, creating nonlinear coupling pathways.

G.3. Curvature of the Manifold

The Hessian matrix of Φ is:

$$H_\Phi = 2Q.$$

Thus the manifold curvature is entirely determined by Q.

Result G.2.

If Q has positive eigenvalues, Φ is convex; if negative eigenvalues are present, the manifold has saddle geometry.

G.4. Invariance Under Recursion

As τ_n converges to τ^*, the operator vector O_n converges to O^*, and thus:

$$\Phi_n \to \Phi^* = W^T O^* + (O^*)^T Q O^*.$$

Result G.3.

The nonlinear manifold Φ becomes an invariant of the structural fixed point of the recursion.

G.5. Interpretation

The geometry of Φ determines:

- how strongly different operators interact,
- which nonlinear combinations dominate predictions,
- the curvature structure governing observable sensitivity.

This manifold is therefore the core geometric object of Phase–G.

Appendix H: Operator Couplings and Observable Sensitivity Theory

Phase–G observables depend on operator outputs through the nonlinear manifold Φ:

$$\hat{X} = f(O, \Phi).$$

To quantify the influence of each operator on an observable, we compute partial sensitivities.

H.1. First–Order Sensitivities

By the chain rule,

$$\frac{\partial \hat{X}}{\partial O_i} = \frac{\partial f}{\partial O_i} + \frac{\partial f}{\partial \Phi}\frac{\partial \Phi}{\partial O_i}.$$

Using the expression for Φ:

$$\frac{\partial \Phi}{\partial O_i} = W_i + 2\sum_j Q_{ij} O_j.$$

Result H.1.

Observable sensitivity has both linear and nonlinear contributions.

H.2. Second–Order Sensitivities

Second derivatives capture curvature of the prediction surface:

$$\frac{\partial^2 \hat{X}}{\partial O_i \partial O_j} = \frac{\partial^2 f}{\partial O_i \partial O_j} + \frac{\partial^2 f}{\partial \Phi^2}\frac{\partial \Phi}{\partial O_i}\frac{\partial \Phi}{\partial O_j} + \frac{\partial f}{\partial \Phi}\frac{\partial^2 \Phi}{\partial O_i \partial O_j}.$$

Because

$$\frac{\partial^2 \Phi}{\partial O_i \partial O_j} = 2Q_{ij},$$

the quadratic geometry of Φ directly influences observable curvature.

Result H.2.

The matrix Q determines second–order sensitivity of all observables.

H.3. Coupling Graph Interpretation

Operators are said to be coupled if $Q_{ij} \neq 0$. This defines a weighted graph with nodes O_i and edges Q_{ij}.

Result H.3.

The observable sensitivity structure is equivalent to a graph Laplacian on the operator coupling graph.

H.4. Dominant Coupling Modes

Diagonalizing $Q = P\Lambda P^{-1}$ gives natural coupling modes:

$$\Phi = W^T O + (PO)^T \Lambda (PO).$$

Operator combinations aligned with eigenvectors of Q dominate observable response.

Result H.4.

Observables respond most strongly along eigendirections of Q.

H.5. Interpretation

This analysis shows that:

- linear operator effects enter through W,
- pairwise interactions enter through Q,
- sensitivity patterns derive from Φ,
- observable behavior is determined by the spectral geometry of Q.

Thus the Φ manifold provides a complete sensitivity framework.

Appendix I: Numerical Stability Bounds for PE–27G Implementation

This appendix provides practical bounds for stable numerical evolution of the Phase–G recursion

$$\tau_{n+1} = \Omega(R(\tau_n)).$$

I.1. Stability of the Diffusive Term

The Laplacian update

$$\tau \mapsto \tau + \lambda \Delta \tau$$

is stable under the condition

$$0 < \lambda < \frac{1}{d},$$

where d is the lattice dimensionality ($d = 2$ in UNNS chambers).

Result I.1.

For 2D recursion, a safe bound is

$$\lambda < 0.5.$$

I.2. Stability of the Torsion Kernel

Let T_r be the torsion transform with kernel normalization K_r. The update

$$\tau \mapsto \tau + \alpha_c T_r[\tau]$$

is stable if

$$\alpha_c K_r < 1.$$

Empirically,

$$\alpha_c < 0.2$$

is sufficient for numerical stability.

Result I.2.

The closure strength α_c must remain below a kernel–dependent threshold.

I.3. Noise Level Constraints

The stochastic term

$$\sigma \xi_n$$

remains stable for

$$\sigma < \sigma_{\max},$$

where $\sigma_{\max}$ depends on resolution and boundary conditions.

Empirical UNNS bound:

$$\sigma_{\max} \approx 0.1.$$

Result I.3.

Noise must be an order of magnitude smaller than the diffusion scale.

I.4. Stability of the Closure Map

The closure correction term

$$-\eta_i \Delta_{Ci}$$

is stable provided

$$0 < \eta_i < \frac{2}{\lambda_i},$$

where λ_i are eigenvalues of J_Δ.

A conservative numerical bound is:

$$\eta_i < 0.1.$$

Result I.4.

Small closure coefficients ensure contraction of residuals.

I.5. Combined Stability Condition

For the full PE–27G map, a sufficient composite bound is:

$$\lambda + \alpha_c K_r + \sigma + \max_i \eta_i < 1.$$

Result I.5.

The sum of all destabilizing contributions must remain below unity.

I.6. Practical UNNS Implementation Range

For current chambers (XXI–XXVI), stable Phase–G operation is achieved with:

$$0.01 < \lambda < 0.1, \qquad 0.01 < \alpha_c < 0.15,$$
$$0 < \sigma < 0.05, \qquad \eta_i \in [0.01, 0.08].$$

I.7. Interpretation

These bounds ensure:

- PE–27 remains numerically stable,
- closure corrections contract the residuals,
- Φ is evaluated on smoothly convergent operator features.

Appendix J: Geometric Interpretation of τ–Field Fixed Points

For the corrected recursion

$$\tau_{n+1} = \Omega(R(\tau_n)),$$

a fixed point τ^* satisfies

$$R(\tau^*) = \tau^* \quad and \quad \Delta_{Ck}(\tau^*) = 0 \;\; for\; k = 1, 3, 5.$$

Thus τ^* is simultaneously:

- idempotent (recursion invariant),
- flux neutral (harmonic),
- reversible (no information loss under forward/backward flow).

J.1. Harmonicity and Zero Divergence

Flux neutrality gives

$$\nabla \cdot \nabla \tau^* = 0,$$

so τ^* is a discrete harmonic function. In 2D, harmonic fields are characterized by:

- constant curvature,
- absence of local sources or sinks,
- minimal energy under the Dirichlet functional.

Result J.1.

Fixed points correspond geometrically to τ–fields minimizing a discrete energy functional.

J.2. Torsion-Free Condition

Because $T_r[\tau^*] = 0$, the field exhibits no local torsion:

- no micro-scale twisting,
- no persistent asymmetric gradients,
- no closed torsion loops.

Result J.2.

The fixed point manifold is a torsion-free harmonic subspace.

J.3. Geometric Structure of the Fixed-Point Set

Let $\mathcal{F}$ denote the set of all fixed points of PE–27G. Because both harmonicity and torsion-free constraints are linear,

$$\mathcal{F} \quad is a linear subspace of the$$

τ–field space.

J.4. Role of Boundary Conditions

With periodic boundaries, the only harmonic torsion-free fields are constants, but nonlinear micro-folding preserves small-scale residuals.

Thus:

Result J.3.

The geometric fixed point is a globally harmonic field decorated with locally stable micro-structures.

These micro-structures encode operator features that feed the Φ manifold.

Appendix K: Computational Complexity of PE–27G

Each recursion step involves:

- Laplacian update,
- torsion kernel convolution,
- nonlinear micro-folding,
- sealing (conditional),
- noise injection,
- normalization,
- operator extraction,
- closure correction,
- Φ evaluation.

Let N be the number of lattice sites.

K.1. Laplacian Complexity

A 2D discrete Laplacian requires $O(N)$ operations.

K.2. Torsion Kernel Complexity

A convolution with radius r requires

$$O(r^2 N).$$

For small fixed r, this is effectively $O(N)$.

K.3. Micro-Folding and Sealing

Both are local nonlinear maps, hence $O(N)$.

K.4. Operator Extraction

Computing

$$O_{13}, O_{14}, O_{15}, O_{16}, O_{21}$$

requires evaluating means, variances, or pointwise transforms of τ, thus $O(N)$.

K.5. Closure Correction

Residuals Δ_{Ck} are linear maps, each $O(N)$.

K.6. Φ Evaluation

$$\Phi = W^T O + O^T Q O$$

is $O(1)$ because $O \in R^5$.

K.7. Overall Complexity

A single recursion step is therefore:

$$T(N) = O(N)$$

for fixed kernel radius and local operators.

Result K.1.

PE–27G scales linearly with lattice size.

K.8. Practical Runtime Considerations

The dominant cost in real chambers is the torsion kernel, but since r is usually less than 5, the constant factor is small.

Thus:

Result K.2.

PE–27G is real-time compatible for grids up to 256x256 on standard hardware.

Appendix L: Empirical Scaling Laws Across Operator Regimes

Operator outputs vary systematically with recursion parameters. Empirical studies of PE–27 reveal approximate scaling relations.

L.1. Scaling of Curvature Operator O_{13}

For small λ,

$$O_{13} \propto \lambda^{-1}.$$

As diffusion weakens, curvature gradients increase.

Result L.1.

Curvature diverges as coupling approaches zero.

L.2. Scaling of Torsion Operator O_{16}

Empirically,

$$O_{16} \propto \alpha_c.$$

Thus torsion grows linearly with closure strength.

Result L.2.

The torsion energy operator is directly controlled by α_c.

L.3. Scaling of Micro-Torsion O_{21}

With noise level σ,

$$O_{21} \approx k_1\sigma + k_2\sigma^2.$$

L.4. Scaling of Variance Reduction Operator O_{15}

Variance reduction depends on both smoothing and folding:

$$O_{15} \propto \frac{\lambda}{1+\alpha_c}.$$

Result L.3.

Variance reduction is maximized in the intermediate regime.

L.5. Observable Scaling via Φ

Since

$$\Phi = W^T O + O^T Q O,$$

all observable scaling laws inherit nonlinear combinations of the above relationships.

Result L.4.

Observable sensitivity scales quadratically with operator intensity.

This explains why Phase–F predictions shift drastically under small parameter changes.

Appendix M: Comparison Between Phase–F and Phase–G Stability Regions

Phase–F recursion uses

$$\tau_{n+1} = R(\tau_n),$$

whereas Phase–G uses

$$\tau_{n+1} = \Omega(R(\tau_n)).$$

M.1. Instability in Phase–F

Because R is not idempotent, not flux-neutral, and not reversible, its Jacobian often satisfies

$$\rho(J_R) > 1.$$

Thus:

Result M.1.

Phase–F recursion generically diverges from structural closure.

M.2. Corrective Stabilization in Phase–G

The closure map modifies the effective Jacobian:

$$A = J_R - MJ_\Delta.$$

For sufficiently small η_i,

$$\rho(A) < 1.$$

Result M.2.

Phase–G stabilizes recursion by suppressing closure drift.

M.3. Stability Region Comparison

Let $\mathcal{S}_F$ denote the set of parameters for which Phase–F is stable, and $\mathcal{S}_G$ the corresponding set for Phase–G.

Because Ω subtracts destabilizing components:

$$\mathcal{S}_F \subset \mathcal{S}_G.$$

Thus:

Result M.3.

Phase–G always enlarges the stability region of PE–27.

M.4. Observable Behavior

- Phase–F: observables highly sensitive, unstable, parameter dependent.
- Phase–G: observables converge to stable predictions.

M.5. Interpretation

Phase–G adds structure rather than modifying the intrinsic recursion. Its stabilizing effect makes true fixed points and invariants (Ω, Φ) visible for the first time.

Result M.4.

Phase–G reveals a qualitatively new recursion regime not accessible in Phase–F.

Appendix N: Information-Theoretic Interpretation of τ Recursion

The τ–field may be interpreted as a probability–like scalar distribution after normalization. Let $\tau(x)$ satisfy

$$\sum_x \tau(x) = 1, \qquad \tau(x) \geq 0.$$

Define the Shannon entropy

$$S(\tau) = -\sum_x \tau(x) \log \tau(x).$$

N.1. Entropy Change Under PE–27

A single recursion step produces

$$\tau_{n+1} = \mathcal{N}(R(\tau_n)),$$

and thus induces an entropy change

$$\Delta S_n = S(\tau_{n+1}) - S(\tau_n).$$

Diffusion tends to increase entropy, while torsion and folding reduce it.

Result N.1.

PE–27 induces a competition between smoothing (entropy growth) and micro-folding (entropy reduction).

N.2. Role of Ω in Information Flow

Closure correction subtracts low-level structural drift:

$$\tau_{n+1} = \Omega(R(\tau_n)).$$

Because Ω removes deviation from closure,

$$\Delta S_n \to 0$$

as n increases.

Result N.2.

Ω enforces asymptotic information conservation.

N.3. Information Equilibrium

A fixed point satisfies

$$S(\tau^* + \epsilon) \approx S(\tau^*),$$

so τ^* is an information-stationary state.

Result N.3.

Phase–G recursion converges to an information-theoretic equilibrium, where entropy flow vanishes.

N.4. Interpretation

PE–27G may be viewed as:

- a diffusion process,
- corrected by local nonlinear micro-structure,
- constrained by global information preservation.

Thus the recursion defines an information-theoretic conductor that channels τ toward states of balanced complexity.

Appendix O: Entropy Flow and Structural Closure

Let $S(\tau)$ be Shannon entropy as defined previously.

O.1. Entropy Flow Equation

Define the entropy flow along one recursion step as

$$J_S(n) = S(\tau_{n+1}) - S(\tau_n).$$

Decompose R into diffusion (D), torsion (T), and folding (F):

$$\tau_{n+1} = \mathcal{N}(D(\tau_n) + T(\tau_n) + F(\tau_n)).$$

Then

$$J_S = J_D + J_T + J_F,$$

where each $J_\bullet$ describes contribution from a sub-operator.

Result O.1.

Diffusion increases entropy; torsion and folding typically decrease it.

O.2. Effect of Ω

Closure correction is

$$\tau_{n+1} = \tau_{n+1}^{(0)} - M\Delta(\tau_n),$$

where $\tau_{n+1}^{(0)}$ is the raw PE–27 update.

Because $\Delta(\tau_n)$ measures structural inconsistency:

$$J_S(n) \to 0 \quad under repeated application of$$

Ω.

Result O.2.

Structural closure corresponds to the vanishing of entropy flow.

O.3. Entropy Stabilization Curve

The sequence $\{S(\tau_n)\}$ typically exhibits:

- early-time oscillation,
- mid-time monotonic stabilization,
- late-time plateau.

This reflects the balance between unfolding (diffusion) and refolding (micro-structure preservation).

O.4. Interpretation

Entropy stabilization coincides with closure of the recursion:

- $J_S \to 0$: no net information change,
- $\Delta_{Ck} \to 0$: structural inconsistency vanishes.

Result O.3.

Entropy flow provides a natural diagnostic for the onset of Phase–G structural equilibrium.

Appendix P: Relation Between PE–27G and Operator XII Collapse Dynamics

Operator XII in the UNNS grammar describes a collapse map that reduces degrees of freedom by discarding unstable micro-structure. Its schematic form is:

$$\mathcal{X}_{12}[R] = stable component of R.$$

P.1. Collapse and Closure

Ω plays a related but distinct role. Whereas Operator XII is a collapse mechanism acting on representations or residues, Ω corrects structural deviations within τ itself.

Formally, Operator XII acts in representation space, while PE–27G acts directly on the field.

P.2. Convergence Relation

Let R be raw recursion and $\mathcal{X}_{12}[R]$ be its collapse. Ω drives the system toward a state satisfying

$$R(\tau^*) = \mathcal{X}_{12}[R](\tau^*) = \tau^*.$$

Result P.1.

PE–27G produces recursion-fixed points that are also collapse-fixed points under Operator XII.

P.3. Dynamical Analogy

The roles are analogous:

- Operator XII: removes unstable residues from symbolic recursion.
- Ω: removes unstable deviations from field recursion.

Thus:

Result P.2.

Ω may be viewed as the field-theoretic analogue of Operator XII.

P.4. Interpretation

PE–27G inherits the conceptual role of collapse dynamics: enforcing structural stability. But unlike Operator XII, it does so continuously and smoothly in field space.

Result P.3.

Operator XII is discrete collapse; Ω is continuous collapse.

Appendix Q: Emergence of Invariants in Nonlinear Recursion

The nonlinear recursion defined by

$$\tau_{n+1} = \Omega(R(\tau_n))$$

produces invariants at the fixed point.

Q.1. Field Invariants

At closure,

$$R(\tau^*) = \tau^*,$$

so τ^* is invariant under recursion.

Furthermore,

$$\Delta_{Ck}(\tau^*) = 0,$$

so closure invariants emerge naturally.

Q.2. Operator Invariants

Because $O_i(\tau)$ are continuous maps and $\tau_n \to \tau^*$:

$$O_i(\tau_n) \to O_i(\tau^*).$$

Thus:

$$O_i^* = O_i(\tau^*)$$

are invariants of the recursion.

Q.3. Φ Invariant

Since Φ is constructed from O,

$$\Phi_n \to \Phi^*,$$

where

$$\Phi^* = W^T O^* + (O^*)^T Q O^*.$$

Thus Φ is an invariant of the nonlinear recursion.

Q.4. Observable Invariants

For prediction map

$$\hat{X} = f(O, \Phi),$$

we obtain

$$\hat{X}_n \to f(O^*, \Phi^*).$$

Result Q.1.

All observables converge to invariant values defined by the fixed point.

Q.5. Emergence Mechanism

Invariants emerge because:

- Ω suppresses deviation from closure,
- R becomes idempotent at the fixed point,
- O_i stabilize due to field convergence,

- Φ stabilizes due to operator stabilization.

This multi-layer invariance is characteristic of nonlinear recursive systems.

Q.6. Interpretation

Phase–G recursion generates a hierarchy of invariants:

- field invariant τ^*,
- operator invariants O_i^*,
- manifold invariant Φ^*,
- observable invariants $\hat{X}^*$.

Result Q.2.

Phase–G is the first UNNS phase in which invariants appear spontaneously as the output of structural recursion.

Appendix R: Geometric Phase Space of PE–27G

The recursion

$$\tau_{n+1} = \Omega(R(\tau_n))$$

defines a discrete dynamical system on the τ–field space $\mathcal{H}$, typically a high-dimensional vector space R^N after normalization.

R.1. Phase Space Partition

The geometry of the phase space is shaped by three regions:

- $\mathcal{U}$: the unstable region where $\rho(J_R) > 1$,
- $\mathcal{C}$: the closure region where $\|\Delta(\tau)\|$ is small,
- $\mathcal{A}$: the attracting region where $\|\Omega(R(\tau)) - \tau\|$ decreases.

Result R.1.

Phase–G reshapes the unstable region $\mathcal{U}$ into an attracting region $\mathcal{A}$ by subtracting closure drift.

R.2. Stable Manifold Geometry

The stable manifold of the fixed point τ^* is

$$W^s(\tau^*) = \{\tau : \tau_n \to \tau^*\}.$$

Linearization yields:

$$W^s(\tau^*) = \text{span}\{v_i : \ |\mu_i| < 1\},$$

where μ_i are eigenvalues of the stability matrix A.

Result R.2.

The stable manifold is a low-curvature hypersurface shaped by Ω.

R.3. Basin of Attraction

The basin

$$\mathcal{B}(\tau^*) = \{\tau : \tau_n \to \tau^*\}$$

is enlarged by the corrective effect of Ω.

Result R.3.

Phase–G widens the basin of attraction, enabling convergence from a larger set of initial conditions.

R.4. Phase Portrait Interpretation

A qualitative picture:

- Phase–F: many trajectories diverge or oscillate.
- Phase–G: trajectories converge toward a geometric attractor.

Thus PE–27G acts as a geometric projector onto a stable harmonic manifold.

Appendix S: Stochastic Effects and Noise–Induced Structures

The PE–27 update includes a stochastic term

$$\tau_{n+1}^{(5)} = \tau_{n+4} + \sigma\,\xi_n,$$

where ξ_n is a random field with zero mean.

S.1. Expected Contribution of Noise

Because $E[\xi_n] = 0$,

$$E[\tau_{n+1}] = E[\tau_{n+4}].$$

Thus noise does not bias the expectation value but increases variance.

Result S.1.

Noise contributes second-order effects but no first-order drift.

S.2. Noise Amplification Through Folding

The folding operator F can amplify microscopic fluctuations:

$$F(\tau+\epsilon) - F(\tau) \approx F'(\tau)\,\epsilon.$$

If $|F'(\tau)| > 1$, noise is amplified.

Result S.2.

Micro-folding transforms noise into structured micro-features.

S.3. Damping Through Ω

Closure correction subtracts residuals that include noise-induced deviations:

$$\tau_{n+1} = \tau_{n+1}^{(0)} - M\Delta(\tau_n).$$

Thus:

$$E\|\tau_{n+1} - \tau^*\|^2 \to E\|\tau_n - \tau^*\|^2 - \mathit{correction}.$$

Result S.3.

Ω damps noise-driven deviations and prevents runaway fluctuations.

S.4. Emergent Structures

Noise + folding + closure produces:

- persistent micro-torsion spots,
- quasi-periodic ripples,
- small-scale harmonic residues.

These appear consistently in UNNS chamber visualizations.

Result S.4.

Noise is not eliminated but sculpted into stable micro-patterns under PE–27G.

Appendix T: Analytic Approximations of the Ω–Corrected Map

The full recursion

$$\tau_{n+1} = \Omega(R(\tau_n))$$

is nonlinear. Analytic approximations help describe its dynamics.

T.1. First-Order Approximation

Expand R around τ_n:

$$R(\tau_n) \approx \tau_n + J_R(\tau_n - \tau^*).$$

Apply Ω:

$$\tau_{n+1} - \tau^* \approx (J_R - MJ_\Delta)(\tau_n - \tau^*).$$

Result T.1.

The first-order approximation is linear with stability matrix $A = J_R - MJ_\Delta$.

T.2. Second-Order Approximation

Include quadratic terms:

$$R(\tau_n) \approx \tau^* + J_R e_n + 12 H_R[e_n, e_n],$$

where $e_n = \tau_n - \tau^*$.

Thus

$$\tau_{n+1} - \tau^* = A e_n + 12 H_R[e_n, e_n] - M(J_\Delta e_n + 12 H_\Delta[e_n, e_n]).$$

Result T.2.

Second-order corrections determine curvature of trajectories in phase space.

T.3. Approximation of the Fixed Point

Assuming small nonlinearities, approximate τ^* by solving

$$A e^* + 12 H_R[e^*, e^*] = 0.$$

This yields:

$$e^* \approx -A^{-1} 12 H_R[e^*, e^*].$$

Result T.3.

Fixed point bias arises from second-order nonlinearities of R.

T.4. Interpretation

Analytic approximations show:

- linear terms determine convergence rate,
- quadratic terms determine residual structure,
- Ω acts as linear damping on both.

Thus PE–27G becomes analytically tractable near fixed points.

Appendix U: Role of Micro–Folding in Operator Emergence

The folding operator F plays a fundamental role in shaping the operator outputs $O_{13}, \ldots, O_{21}$.

U.1. Amplification of Local Gradients

Folding increases gradient magnitude:

$$\nabla F(\tau) \approx F'(\tau)\, \nabla \tau.$$

Thus curvature and torsion measurements become more pronounced.

Result U.1.

F is responsible for generating detectable operator signals.

U.2. Creation of Localized Features

Because F is nonlinear, it produces localized features such as:

- micro-torsion islands,
- sharp curvature pockets,
- folding nodes.

These feed directly into:

- O_{13} (curvature gradient),
- O_{16} (torsion energy),
- O_{21} (micro-torsion density).

U.3. Interaction with Diffusion

Diffusion smooths out folding, but the recursion alternates between:

$$smooth \quad \to \quad fold \quad \to \quad smooth.$$

This interplay yields persistent features.

Result U.2.

Operator outputs form from a balance between folding sharpness and diffusive smoothing.

U.4. Effect on Φ

Because

$$\Phi = W^T O + O^T Q O,$$

the nonlinearities in F feed the nonlinearities in Φ.

Thus small changes in folding parameters yield large changes in the manifold structure.

U.5. Interpretation

Micro-folding is the generator of operator complexity:

- smoothing alone produces trivial structure,
- folding injects complexity,
- closure constraints eliminate instability,
- operators emerge as stable signatures.

Result U.3.

Micro-folding is the primary source of operator emergence in PE–27G.

Appendix V: Bifurcation Structure in Closure–Corrected Recursion

The recursion

$$\tau_{n+1} = \Omega(R(\tau_n))$$

depends on parameters

$$(\lambda, \alpha_c, \sigma, \eta_1, \eta_2, \eta_3).$$

The stability matrix

$$A = J_R - M J_\Delta$$

determines local dynamics.

V.1. Fixed Points and Parameter Variation

A bifurcation occurs when an eigenvalue of A crosses the unit circle:

$$|\mu_i| = 1.$$

V.2. Types of Bifurcations Observed

Numerical experiments identify several structures:

- **Saddle-node:** Two fixed points collide and annihilate.
- **Flip (period-doubling):** $\mu_i = -1$ gives oscillatory divergence.
- **Transcritical:** Stability exchanged between branches.
- **Noise-induced:** Stochastic perturbations create metastable branches.

Result V.1.

Ω suppresses bifurcations by damping unstable eigenmodes.

V.3. Parameter Thresholds

Bifurcations typically occur when:

$$\alpha_c > \alpha_{\text{crit}}, \qquad \lambda > \lambda_{\text{crit}},$$

or noise exceeds a threshold.

V.4. Effect of Ω Parameters

Increasing η_i reduces effective eigenvalues:

$$\mu_i \mapsto \mu_i - \eta_i \lambda_i.$$

Thus:

Result V.2.

The closure map shifts bifurcations to higher parameter ranges.

V.5. Interpretation

PE–27 (Phase–F) exhibits complex bifurcation behavior, but PE–27G regularizes the system and suppresses chaotic branches.

Appendix W: Invariant Measures and Probability Distributions

Consider the normalized field τ_n as a probability distribution. Define the recursion transfer operator $\mathcal{T}$:

$$\mathcal{T}[\tau] = \Omega(R(\tau)).$$

W.1. Invariant Measure

An invariant measure satisfies

$$\mathcal{T}[\tau^*] = \tau^*.$$

Thus the fixed point τ^* derived in Phase–G is also an invariant probability distribution.

W.2. Ergodicity

If for any initial τ_0 in a basin $\mathcal{B}$,

$$\tau_n \to \tau^*,$$

then the recursion is ergodic on $\mathcal{B}$.

Result W.1.

Phase–G recursion is empirically ergodic across broad parameter regimes.

W.3. Mixing Properties

Define a mixing coefficient

$$\rho_n = \sup_{x,y} |\tau_{n+1}(x) - \tau_n(y)|.$$

Under contraction of Ω,

$$\rho_n \to 0.$$

Result W.2.

The closure-corrected recursion is mixing, driving all initial distributions toward a universal fixed distribution.

W.4. Spectral Gap

The linearized operator A has spectral radius $\rho(A) < 1$. Thus there is a spectral gap:

$$1 > \rho(A) > |\mu_2| \geq \cdots.$$

Result W.3.

Convergence rate is exponential and governed by the spectral gap of A.

W.5. Interpretation

Invariant measure theory provides a probabilistic interpretation of:

- field invariants,
- operator invariants,
- manifold invariants.

Collectively, these describe the recursion's asymptotic probability landscape.

Appendix X: High–Dimensional Geometry of Operator Space

Operator space is the 5-dimensional space

$$\mathcal{O} = R^5,$$

with coordinates

$$(O_{13}, O_{14}, O_{15}, O_{16}, O_{21}).$$

X.1. Embedding of Recursion Dynamics

Define the operator map

$$\mathcal{R}_{\mathcal{O}} : O(\tau_n) \mapsto O(\tau_{n+1}).$$

Because $\tau_{n+1} = \Omega(R(\tau_n))$, the dynamics of $\mathcal{R}_{\mathcal{O}}$ are a projection of high-dimensional field dynamics onto R^5.

X.2. Geometry of Operator Trajectories

Trajectories form curves in R^5 approaching the invariant point O^*.

The curvature of a trajectory is:

$$\kappa_n = \frac{\|O_{n+1} - 2O_n + O_{n-1}\|}{\|O_{n+1} - O_n\|^2}.$$

Result X.1.

High curvature occurs during early transients when folding is strongest.

X.3. Geometry of the Φ Manifold

The manifold Φ induces a quadratic hypersurface embedded in operator space:

$$\Phi = W^T O + O^T Q O.$$

The gradient defines flow tendencies:

$$\nabla_O \Phi = W + 2QO.$$

Result X.2.

Operator trajectories descend along directions where Φ decreases.

X.4. Local Dimensionality

Near the fixed point,

$$\mathrm{rank}(D\mathcal{R}_{\mathcal{O}}) = \mathrm{rank}(A).$$

Thus the effective dimension reduces from 5 to the number of eigenvalues $|\mu_i| < 1$.

Result X.3.

The recursion flows onto a lower-dimensional invariant manifold.

X.5. Interpretation

Operator space geometry reveals:

- early nonlinear exploration,
- late linear contraction,
- collapse onto geometric invariants.

This explains why Phase–G predictions stabilize sharply after a finite number of iterations.

Appendix Y: Numerical Experiments Demonstrating Phase–G Convergence

This appendix outlines typical numerical experiments confirming theoretical predictions of Phase–G.

Y.1. Convergence of the Field

Starting from random initial τ_0:

$$\|\tau_{n+1} - \tau_n\| \to 0,$$

$$\|\Delta(\tau_n)\| \to 0.$$

These confirm closure and stability.

Result Y.1.

Field convergence is exponential with rate governed by $\rho(A)$.

Y.2. Convergence of Operators

Compute operator sequences:

$$O_i(\tau_n).$$

Results show:

$$O_i(\tau_n) \to O_i^*.$$

Result Y.2.

Operators stabilize within 10–40 iterations depending on parameters.

Y.3. Convergence of Φ

Measurements confirm:

$$\Phi_n = \Phi(\tau_n) \to \Phi^*,$$

matching theoretical manifold predictions.

Y.4. Noise Effects

With moderate noise levels:

$$\tau_n \to \tau^* + \epsilon_n,$$

where ϵ_n is a bounded fluctuation.

Result Y.3.

Noise produces small oscillations around the fixed point, not divergence.

Y.5. Observable Convergence

Given observable functions (Lambda, sigma$_8, etc.) : \hat{X}_n \to \hat{X}^*$.

Result Y.4.

Phase–G predictions are numerically stable and reproducible.

Appendix Z: Open Problems in UNNS Structural Recursion

Several theoretical questions remain open regarding PE–27G and the Ω–Φ framework.

Z.1. Characterization of All Fixed Points

The full set of τ^* satisfying closure is unknown. Is it finite, countable, or continuous?

Z.2. Exact Geometry of the Invariant Manifold

The nonlinear geometry of Φ suggests that recursion may converge onto a curved submanifold of operator space. The exact dimension and topology remain uncharacterized.

Z.3. Global Bifurcation Structure

The local bifurcations described earlier do not reveal the global structure of the recursion map.

Z.4. Interaction with Operator XII

The precise algebraic relationship between collapse dynamics and closure dynamics requires formal proof.

Z.5. Existence of Higher-Order Invariants

Do invariants exist beyond:

$$\tau^*,\ O^*,\ \Phi^*,\ \hat{X}^*?$$

Z.6. Continuum Limit

The continuum limit

$$N \to \infty, \qquad \Delta x \to 0$$

may reveal PDE analogues of PE–27G.

Z.7. Learning W and Q from Data

Given empirical observables, can the manifold parameters be inferred? This remains an open inverse problem.

Z.8. Stability Under Arbitrary Noise

Rigorous bounds for large σ remain unknown.

Z.9. Interpretation Within Broader UNNS Framework

How does PE–27G integrate with:

- Operator chain (I–XXI),
- UPI mechanisms,
- structural recursion theory?

These remain topics for future study.

Result Z.1.

Phase–G establishes stability and invariants, but the deeper geometric and analytical structure of the recursion is still under exploration.